Beauty in Mathematics

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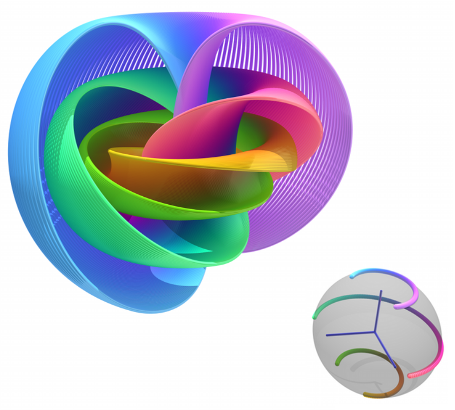
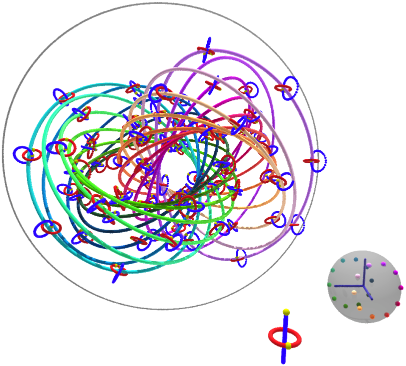
The Ohio State University
NEWARK

Explain spheres in higher dimensions



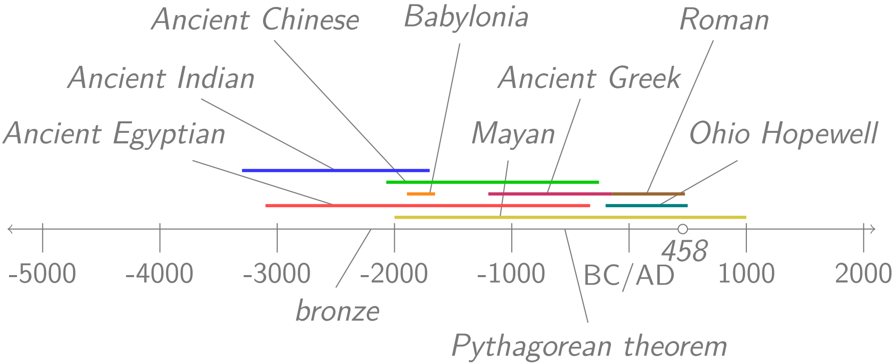


- Two applications of counting
- Thinking about higher dimensions
- Video clips

<the concept of nothing>

0

- number of nothing
- placeholder for place-value numbers
  10, 100, 1000



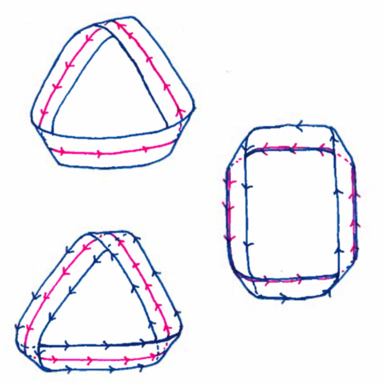
- 0 as a placeholder and independent number: India, 458 AD.
- Appeared in Europe in 1202 place-value arithmetic book.

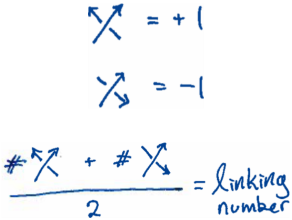
0

- number of nothing
- placeholder for place-value numbers
  10, 100, 1000

$\textit{even} + \textit{even} = \textit{even}$	$0 + 0 = 0$
$\textit{odd} + \textit{even} = \textit{odd}$	$1 + 0 = 1$
$\textit{even} + \textit{odd} = \textit{odd}$	$0 + 1 = 1$
$\textit{odd} + \textit{odd} = \textit{odd}$	$1 + 1 = 0$

Counting bands





THEOREM If linking number is NOT zero, then loops ARE linked.

WARNING It can happen that loops are linked, but their linking number is zero. And more complicated things can happen with 3 or more loops.

Counting faces

	V	E	F	$V - E + F$
	8	12	6	2
	4	6	4	2
	6	12	8	2
	12	30	20	2
	12	18	8	2
	62	180	120	2
	56	180	120	-4

Euler characteristic $= \chi = V - E + F_2 - F_3 + F_4 - F_5 + \cdots$

$\chi($  $) = 0$

$\chi($  $) = 1$

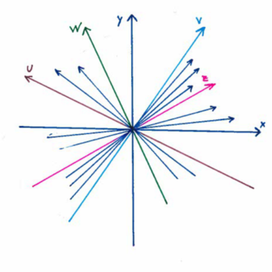
$\chi($  $) = 2$

$\chi($  $) = 0$

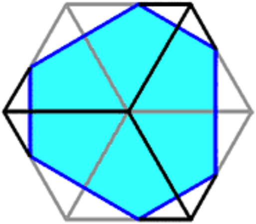
Slices and projections

Ways to understand higher dimensions

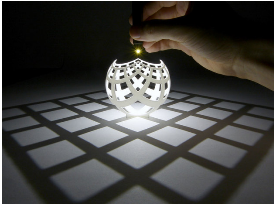
- Gluing pieces together: lower-dimensional intersection
  Euler characteristic is additive
- Coordinates: $(x_1, x_2, x_3, x_4, x_5, \ldots, x_n)$
- Slices: lower-dimensional cross-section
- Projection: push to lower dimension



Coordinates



Cube Slices (link)



Stereographic Projection (link)

Spheres

- the circle $S^1 = \{x^2 + y^2 = 1\}$
- the 2-sphere $S^2 = \{x^2 + y^2 + z^2 = 1\}$
- the n-sphere $S^n = \{x_1^2 + x_2^2 + \cdots + x_{n+1}^2 = 1\}$

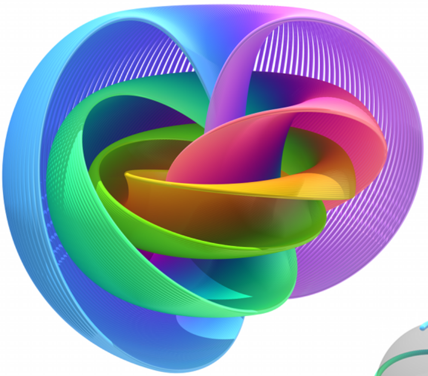
Spherical projections in dimensions $n = 2, 4, 8, 16$. Period.

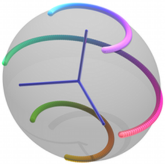
S^{n-1} projects to $S^{n/2}$, with fibers $S^{n/2-1}$.

$$S^{n/2-1} \hookrightarrow S^{n-1} \twoheadrightarrow S^{n/2}$$

We will look at $n = 2, 4, 8$.

The Hopf fibration $S^1 \hookrightarrow S^3 \twoheadrightarrow S^2$

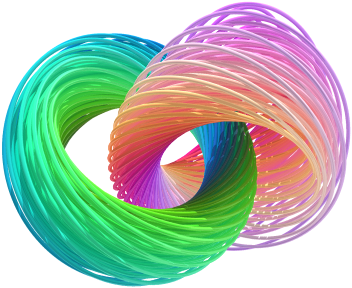




Heinz Hopf, right
(1894 – 1971).

- Every point on S^2 has a S^1 above it, called its *fiber*.
- Each pair of fibers has linking number 1.
- Two linked tubes, joined along their boundary.

The Hopf fibration $S^1 \hookrightarrow S^3 \twoheadrightarrow S^2$



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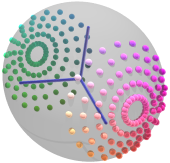

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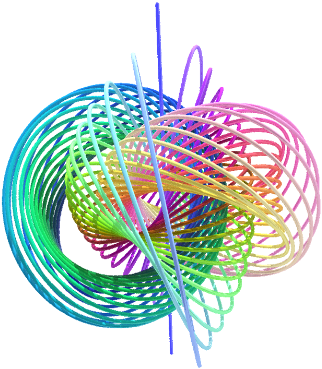

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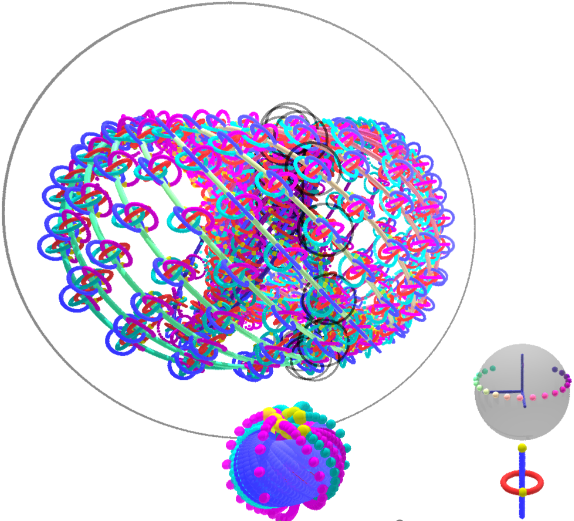
The Hopf fibration $S^3 \hookrightarrow S^7 \twoheadrightarrow S^4$



John Milnor, right (born 1931).

- Every point on S^4 has a S^3 above it, called its *fiber*.
- Two linked 7-tubes, joined along their boundary.
- Geometrically distinct 7-spheres obtained by different gluings.

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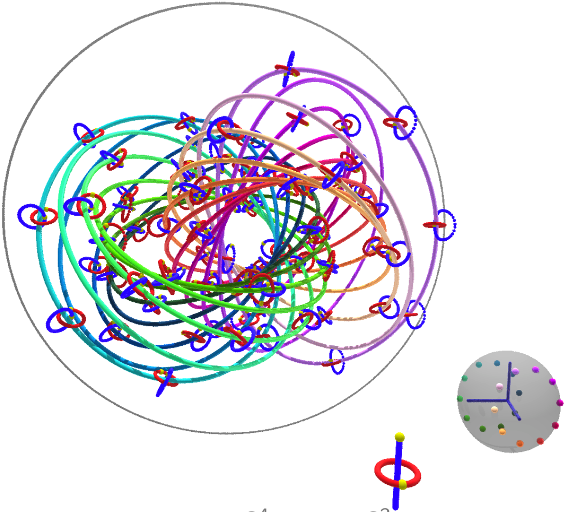

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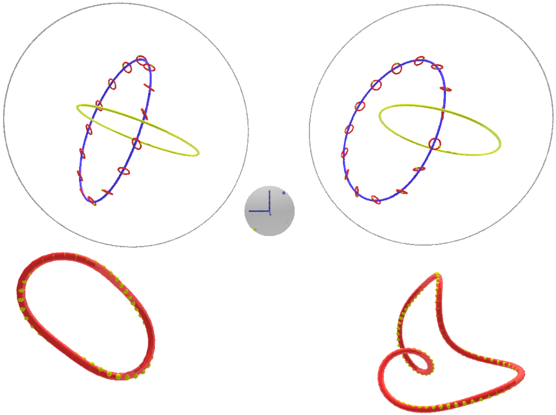

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Movie Time!

<play movies>

3-spheres	`https://www.youtube.com/watch?v=AKotMPGFJYk`
7-spheres	`https://www.youtube.com/watch?v=II-maE5HEj0`

The End

Thank You!

For other resources related to the talk, see `nilesjohnson.net/beauty.html`.