Power operation calculations in elliptic cohomology

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Special session on homotopy theory 2014

Elliptic cohomology and Morava E-theory

Definition (Ando-Hopkins-Strickland '01, Lurie '09)

elliptic cohomology theory $= \left\{ \begin{array}{ll} S, \quad C/S, & E, \\ E^0(*) \cong S, & \operatorname{Spf} E^0(\mathbb{CP}^\infty) \cong \widehat{C} \end{array} \right\}$

Theorem (Goerss-Hopkins-Miller)

$\mathcal{E}$: {formal groups over perfect fields, isos} $\to$ {E_∞-ring spectra}

- $\operatorname{Spf} E^0(\mathbb{CP}^\infty) =$ the univ deformation of a fg F of height n over a perfect field k of char p
- $E_* = \pi_* E \cong \mathbb{W}(k)[\![u_1, \ldots, u_{n-1}]\!][u^{\pm 1}], \quad |u| = 2$

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Goal explore the structure on E_*.

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Power operations for Morava E-theory (height n prime p)

$M = E$-module $\qquad \pi_0 M = [S, M]_S \cong [E, M]_E$

$$\mathbb{P}_E(M) = \bigvee_{i \geq 0} \mathbb{P}_E^i(M) = \bigvee_{i \geq 0} (\underbrace{M \wedge_E \cdots \wedge_E M}_{i\text{-fold}})_{h\Sigma_i}$$

$A =$ commutative E-algebra
$\quad =$ algebra for the monad $\mathbb{P}_E$ with $\mu\colon \mathbb{P}_E(A) \to A$

total power operation $\psi^i\colon \pi_0 A \to \pi_0\left(A^{B\Sigma_i^+}\right)$
$\forall \eta \in \pi_0 \mathbb{P}_E^i(E)$, individual po $Q_\eta\colon \pi_0 A \to \pi_0 A$
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Theorem (Rezk '09, Barthel-Frankland '13)

If $A = K(n)$-local commutative E-algebra, then

$$A_* = \text{graded amplified } L\text{-complete } \Gamma\text{-ring}$$

- Γ = twisted bialgebra over E_0 (Dyer-Lashof algebra)
- $\exists\, Q_0 \in \Gamma$ with $Q_0(x) \equiv x^p \bmod p$ (Frobenius congruence)

Goal make this structure explicit at $n = 2$, $p = 3$.
(Rezk '08, $n = 2$, $p = 2$)

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$\psi\colon C \to C/G$ restricts as $\psi_0\colon C_0 \to C_0$ (3-power Frob)

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Define individual power operations $Q_i \colon E^0 \to E^0$ by

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$$\psi^3_F(h) = h^3 - 27h^2 + 183h - 180 + 186h^{-1} + 1674h^{-2} + \cdots$$

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Future directions

<u>Question</u> Can we get, for **all** p, a uniform presentation of the Dyer-Lashof algebra Γ for Morava E-theory at height 2?

A uniform presentation for Γ/p has been given at ht 2 (Rezk '12).

$$\psi^p\colon\ E^0 \to E^0 B\Sigma_p/I \cong \mathbb{Z}_{p^2}[\![h]\!][\alpha]/\big(w(\alpha)\big) \cong \mathbb{Z}_{p^2}[\![\alpha,\alpha']\!]/(\alpha\alpha' + p)$$

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