

The 2-dimensional stable homotopy hypothesis

OR
Introduction to twice-categorified abelian groups

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What we proved

Theorem: There is an equivalence of homotopy theories

(categories with weak equivalences)

Stable 2-types \simeq Picard 2-categories

w/ stable weak equiv.s

w/ categorical equiv.s

Stable under suspension
- group comp'n of π_0
- abelianization of π_0, π_1 (Eoo str.)

$[\pi_0, \pi_1, \pi_2 \text{ \& } 2 \text{ Postnikov mv'ts}]$

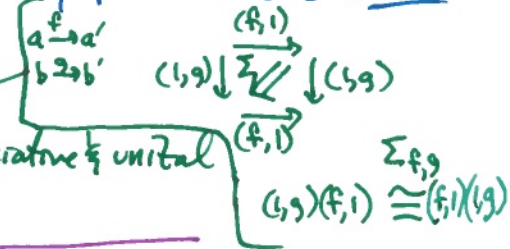
$[\text{Permutative Gray Monoids}]$ w/ obs. \& mor. invertible

What we learned

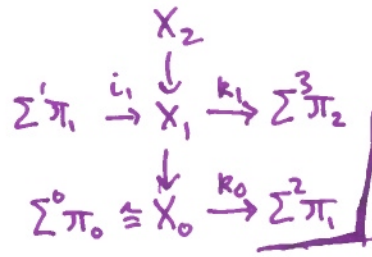
The strictest kind of SM bicategory which encodes fully general homotopy of stable 2-types is monoid w.r.t. Gray product on 2Cat.

PGM: 2-category
• Gray monoid
• permutative

(strictly associative \& unital)



Postnikov tower



Context:

• Stable 0-types \simeq Abelian groups

• Stable 1-types \simeq Picard categories
[grouplike SM groupoids
a.k.a. abelian 2-groups]

$k_0: \pi_0/2 \rightarrow \pi_1 \iff \text{symm } e \simeq x \cdot x^{-1} \simeq x^{-1} \cdot x \simeq e$

$P_1 S \iff \text{Pic}(\text{Ch}(\underline{Ab}))$
obj. $\simeq \mathbb{Z}$
aut $\simeq \mathbb{Z}/2$

• infinite loop space machine for PGM (K-thy paper)

• homotopy theory w/ strict v.s. weak maps (homotopy theories \& adjunctions)

• k_i given by $\Sigma_{f, f'}$ (Postnikov data paper)

• SM braut structure from Eoo operad action (SHH2)

Next Steps

Homotopy theory of Picard 2-categories

- Homological algebra of Picard 1-categories
- 2-categorical group-completion (Quillen's S's)
- homotopy fiber/cofiber via comma constr. (\& Quillen B)

Questions

• $P_2 S \simeq \text{Pic}(\text{Ch}(\underline{\text{Pic}}))$

• $d_3: \ker(k_0) \xrightarrow{?} \text{coker}(k_{i_1})$
we have 2 guesses

2-category of Picard 1-categories

- categorification of \mathbb{Z}
- correct π_0, π_1, π_2 \& k_0, k_{i_1}
- need SM braut str.