

ERRATA

2-DIMENSIONAL CATEGORIES
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This document contains known errors and corrections as of 15 November 2021. If you are aware of further errors, please contact both authors by email:

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Page numbers refer to the published version of the text.

1. PAGE 243, EXPLANATION 6.1.3, ITEM (3)

In the bottom right triangle, the slanted arrow should be $\underline{1}_g$, not $\underline{1}_f$.

2. PAGE 320, LEMMA 8.3.12

In this proof, each instance of the left unitor ℓ should be replaced with the right unitor r because its components

$$r_f : f1_A \xrightarrow{\cong} f \quad \text{and} \quad r_{f'} : f'1_A \xrightarrow{\cong} f'$$

provide the desired natural isomorphism. Moreover, the first and third displays should use single arrows \longrightarrow instead of double arrows \Rightarrow as that is our convention outside of cell diagrams. The specific changes are as follows.

2.1. Sentence 3 should be replaced with the following.

Thus the right unitor defines a natural isomorphism

$$r : e_A \circ \mathcal{Y} \longrightarrow \text{Id}_{\mathbf{B}(A,B)}.$$

2.2. Sentence 5: replace "the left unitor ℓ " with "the right unitor r ".

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2.3. Display 3 should be replaced with the following.

$$\mathcal{Y} \circ e_A \longrightarrow \text{Id}_{\text{Str}(\mathcal{Y}_A, \mathcal{Y}_B)}.$$

3. PAGE 320, COROLLARY 8.3.13

This result and the paragraph immediately before it are incorrect as stated. They should be replaced by the following corrected version.

Examining the proof of Lemma 8.3.12, we note that the modification components constructed there will be identities if \mathbf{B} has trivial unitors and each θ is a strict transformation. Recall from Corollary 8.2.16 that \mathcal{Y} is a 2-functor if \mathbf{B} is a 2-category. Moreover, in that case then each \mathcal{Y}_f is a strict transformation between 2-functors. Thus, we have the following.

Corollary 8.3.13. *If \mathbf{B} is a 2-category, then for each pair of objects A and B in \mathbf{B} , the Yoneda 2-functor \mathcal{Y} provides an isomorphism of categories*

$$\mathcal{Y} : \mathbf{B}(A, B) \xrightarrow{\cong} \text{Strict}(\mathcal{Y}_A, \mathcal{Y}_B)$$

where

$$\text{Strict}(\mathcal{Y}_A, \mathcal{Y}_B) = 2\text{Cat}(\mathbf{B}^{\text{op}}, \text{Cat})(\mathcal{Y}_A, \mathcal{Y}_B)$$

denotes the category of strict transformations and modifications between \mathcal{Y}_A and \mathcal{Y}_B .

The correction to Corollary 8.3.13 also requires corrections to Corollary 8.4.2 on page 328, noted below.

4. PAGE 328, COROLLARY 8.4.2

Make the following changes in the paragraph before Corollary 8.4.2.

- 4.1. In the first sentence, replace “[Corollary 8.3.13](#)” with “[Corollary 8.2.16](#)”.
- 4.2. The second sentence should be changed to reflect the correction to Corollary 8.3.13 above. It should be replaced by the following two sentences.

If \mathbf{B} is a 2-category, then Corollary 8.3.13 shows that \mathcal{Y} induces a local isomorphism to $2\text{Cat}(\mathbf{B}^{\text{op}}, \text{Cat})$, the 2-category consisting of 2-functors from \mathbf{B}^{op} to Cat together with strict transformations and modifications. Therefore, by the 2-categorical Whitehead Theorem 7.5.8, we obtain the following.

- 4.3. Corollary 8.4.2 is incorrect as stated. It should be replaced with the following corrected version.

Corollary 8.4.2. *If \mathbf{B} is a 2-category, let stB_2 denote the essential image of*

$$\mathcal{Y} : \mathbf{B} \longrightarrow 2\text{Cat}(\mathbf{B}^{\text{op}}, \text{Cat}).$$

Then

$$\mathbf{B} \longrightarrow \text{stB}_2$$

is a 2-equivalence.